SOLUTIONS TO PRACTICE FINAL QUESTIONS.
MICHAEL MAGEE
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① $\left(\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 2 \\ 2 & 6 & 4 & 8 & 4 \\ 3 & 9 & 3 & 7 & 5 \end{array}\right) \longrightarrow \begin{array}{c} \begin{array}{cccc} \text{PIV} & \text{FREE} & \text{PIV} & \text{PIV} \end{array} \\ \left(\begin{array}{cccc|c} \boxed{1} & 3 & 1 & 2 & 2 \\ 0 & 0 & \boxed{2} & 4 & 0 \\ 0 & 0 & 0 & \boxed{1} & -1 \end{array}\right) \end{array}$

PARTICULAR SOLUTION: $x_4 = -1$

$2x_3 + 4x_4 = 0 \Rightarrow x_3 = 2$

$x_2 = 0$ (FREE COLUMN).

$x_1 + 3x_2 + x_3 + 2x_4 = 2$

$x_1 = 2$

$\underline{x} = \begin{pmatrix} 2 \\ 0 \\ 2 \\ -1 \end{pmatrix}$

SPECIAL SOLUTION: SET $x_2 = 1$ AND RIGHT HAND COL TO $\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$

$\begin{matrix} 1 & 3 & 1 & 2 & 0 \\ 0 & 0 & 2 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{matrix} \Rightarrow x_4 = 0,\ x_3 = 0,\ x_1 = -3$

SPECIAL SOLUTION $\underline{x}_n = \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}$.

GENERAL SOLUTION: $\begin{pmatrix} 2 \\ 0 \\ 2 \\ -1 \end{pmatrix} + x_2 \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, x_2 in $\mathbb{R}$.

② . SEE #2 PRACTICE MIDTERM.

③ ELIMINATION GIVES

$\begin{matrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix} \xrightarrow{R2 - \frac{1}{2}R_1} \begin{matrix} 2 & 1 & 1 \\ 0 & 3/2 & 1/2 \\ 1 & 1 & 2 \end{matrix} \xrightarrow{R_3 - \frac{1}{2}R_1} \begin{matrix} 2 & 1 & 1 \\ 0 & 3/2 & 1/2 \\ 0 & 1/2 & 3/2 \end{matrix} \xrightarrow{R_3 - \frac{1}{3}R_2} \begin{matrix} 2 & 1 & 1 \\ 0 & 3/2 & 1/2 \\ 0 & 0 & 4/3 \end{matrix} = \underline{U}.$

$\left(\frac{3}{2} - \frac{1}{6} = \frac{4}{3}\right)$

So $U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{3} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$,

$A = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{3} & 1 \end{pmatrix}}_{L} U$

$\underline{L} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{2} & \frac{1}{3} & 1 \end{pmatrix}$

④: SEE MIDTERM PRACTICE Q. 3.

⑤ $\begin{matrix} 1 & 1 & 2 & 4 \\ 1 & 2 & 2 & 5 \\ 1 & 3 & 2 & 6 \end{matrix} \longrightarrow \begin{matrix} 1 & 1 & 2 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 0 & 2 \end{matrix} \longrightarrow \begin{matrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}.$

6. GRAM SCHMIDT

$\underline{a} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}$ $\underline{c} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$.

$\underline{A}^T \underline{b} = 2$ $\quad \underline{A}^T \underline{c} = 6$

$\underline{A}^T \underline{A} = 2$ $\quad \underline{B}^T \underline{c} = -6$

$\underline{B}^T \underline{B} = 6$

$$\underline{A} = \underline{a} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad \underline{B} = \underline{b} - \frac{\underline{A}^T \underline{b}}{\underline{A}^T \underline{A}} \underline{A} = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}.$$

$$\underline{C} = \underline{c} - \frac{\underline{A}^T \underline{c}}{\underline{A}^T \underline{A}} \underline{A} - \frac{\underline{B}^T \underline{c}}{\underline{B}^T \underline{B}} \underline{B} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix} - 3 \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

$$q_1 = \frac{\underline{A}}{\|\underline{A}\|} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad q_2 = \frac{\underline{B}}{\|\underline{B}\|} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \quad q_3 = \frac{\underline{C}}{\|\underline{C}\|} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

7. $$|A - \lambda I| = \begin{vmatrix} 2-\lambda & -1 & 1 \\ 1 & -\lambda & 3 \\ 0 & 0 & 3-\lambda \end{vmatrix} = \text{(BIG FORMULA, OR YOUR FAVORITE WAY OF COMPUTING 3X3 DETERMINANTS)} = (-\lambda)(2-\lambda)(3-\lambda) + (3-\lambda)$$

$$= (3-\lambda)(1 - \lambda(2-\lambda)) = (3-\lambda)(\lambda^2 - 2\lambda + 1) = (3-\lambda)(\lambda - 1)^2.$$

$\lambda = 3$ or 1. EIGENVALUES.

EIGENVECTORS: SOLVE $(A - \lambda I)\underline{v} = \underline{0}$. $\quad \lambda = 3: \begin{pmatrix} -1 & -1 & 1 \\ 1 & -3 & 3 \\ 0 & 0 & 0 \end{pmatrix} \underline{v} = \underline{0}$.

elimination $\rightarrow$ $\begin{matrix} \boxed{-1} & -1 & 1 \\ 0 & \boxed{-4} & 4. \end{matrix}$

one free column; set $x_3 = 1$, then $x_2 = 1$, $x_1 = 0$.

$\lambda = 3: \quad \underline{v} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$.

$\lambda = 1: \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & 3 \\ 0 & 0 & 2 \end{pmatrix} \underline{v} = 0$.

elimination $\rightarrow$ $\begin{matrix} 1 & -1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{matrix} \rightarrow \begin{matrix} \boxed{1} & -1 & 1 \\ 0 & 0 & \boxed{2} \\ 0 & 0 & 0 \end{matrix}$.

$\lambda = 1$ $\quad \underline{v} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

set $x_2 = 1$ (free column) $\Rightarrow x_3 = 0$. $x_1 = 1$.

USE ELIMINATION!

(8)
$$\begin{matrix} 1 & 2 & 1 & 2 \\ -1 & -4 & -6 & 0 \\ 3 & 6 & 6 & 1 \\ 1 & -4 & -14 & 6 \end{matrix} \xrightarrow{\begin{matrix} R_2+R_1 \\ R_3-3R_1 \\ R_4-R_1 \end{matrix}} \begin{matrix} 1 & 2 & 1 & 2 \\ 0 & -2 & -5 & 2 \\ 0 & 0 & 3 & -5 \\ 0 & -6 & -15 & 4 \end{matrix} \xrightarrow{R_3-3R_2} \begin{matrix} 1 & 2 & 1 & 2 \\ 0 & -2 & -5 & 2 \\ 0 & 0 & 3 & -5 \\ 0 & 0 & 0 & -2 \end{matrix}.$$

DETERMINANT UNCHANGED: PRODUCT OF PIVOTS $= 1(-2)3(-2) = 12$.

(9) $|A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = \lambda(\lambda-1) - 1 = \lambda^2 - \lambda - 1$ $\lambda_\pm = \frac{1}{2}(1 \pm \sqrt{5})$.

EVECTORS $\underline{v}_+$ $A - \lambda_+ = \begin{pmatrix} \frac{1}{2} - \frac{\sqrt{5}}{2} & 1 \\ 1 & -\frac{1}{2} - \frac{\sqrt{5}}{2} \end{pmatrix}$. NULLSPACE SPANNED BY $\underline{v}_+ = \begin{pmatrix} 1 \\ \frac{\sqrt{5}}{2} - \frac{1}{2} \end{pmatrix}$

$\underline{v}_-$ $A - \lambda_- = \begin{pmatrix} \frac{1}{2} + \frac{\sqrt{5}}{2} & 1 \\ 1 & -\frac{1}{2} + \sqrt{5}/2 \end{pmatrix}$ $\underline{v}_- = \begin{pmatrix} 1 \\ -\frac{1}{2}(\sqrt{5}+1) \end{pmatrix}$

$\Lambda = \begin{pmatrix} \frac{1}{2}(1+\sqrt{5}) & 0 \\ 0 & \frac{1}{2}(1-\sqrt{5}) \end{pmatrix}$. $S = \begin{pmatrix} 1 & 1 \\ \frac{1}{2}(\sqrt{5}-1) & -\frac{1}{2}(\sqrt{5}+1) \end{pmatrix}$.

$$\det(S) = -\frac{1}{2}(\sqrt{5}+1) - \frac{1}{2}(\sqrt{5}-1) = -\sqrt{5}$$

$\underline{A = S \Lambda S^{-1}}$.

$$S^{-1} = \frac{-1}{\sqrt{5}} \begin{pmatrix} -\frac{1}{2}(\sqrt{5}+1) & -1 \\ \frac{1}{2}(1-\sqrt{5}) & 1 \end{pmatrix}.$$

IT'S POSSIBLE TO DIAGONALIZE A BECAUSE A IS SYMMETRIC.

(10) $|A - \lambda I| = \begin{vmatrix} 3-\lambda & 2 \\ -5 & -3-\lambda \end{vmatrix} = -(3+\lambda)(3-\lambda)+10 = 10 - (9-\lambda^2) = \lambda^2 + 1$.

SO THERE ARE 2 DISTINCT EIGENVALUES $\lambda = \pm i$

SO FOR SOME S, $A = S \Lambda S^{-1}$ $\Lambda = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$ THEN $A^{1024} = S \Lambda^{1024} S^{-1} = S I S^{-1} = \underline{I}$.

11. $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ IS IN JORDAN NORMAL FORM,

WITH A JORDAN BLOCK OF SIZE 3,

SO NOT SIMILAR TO A DIAGONAL MATRIX.

12. $A = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ HAS EIGENVALUES $\lambda = 1,\ \underline{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

$\lambda = 0,\ \underline{v}_0 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

WRITE $\underline{u}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2}\underline{v}_1 + \frac{1}{2}\underline{v}_0$ AS LIN. COMBINATION OF EVECTORS.

THEN $A^k \underline{u}_0 = \frac{1}{2} A^k \underline{v}_1 + \frac{1}{2} A^k \underline{v}_0 = \frac{1}{2}\underline{v}_1$ WHEN $k \geq 1$

BUT $\underline{u}_k = A^k \underline{u}_0$, SO $\underline{u}_k = \frac{1}{2}\underline{v}_1$ WHEN $k \geq 1$ (CONSTANT AFTER $\underline{u}_0$).

13 USE UPPER LEFT MATRICES. (FOR EXAMPLE)

$|(2)| = 2 > 0$ ✓ $\quad \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 3 > 0$ ✓ $\quad \begin{vmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 8+1+1-2-2-2 = 4$ ✓ > 0. (BIG FORMULA)

ALL DETERMINANTS $> 0 \Rightarrow A$ POS. DEF.

14. $A^TA = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 5 \end{pmatrix}$.

$$|A^TA - \lambda I| = \begin{vmatrix} 2-\lambda & 2 \\ 2 & 5-\lambda \end{vmatrix} = (2-\lambda)(5-\lambda) - 4 = \lambda^2 - 7\lambda + 6\lambda = (\lambda-6)(\lambda-1).$$

$\lambda_1 = 6 \quad \lambda_2 = 1$

$\sigma_1 = \sqrt{6} \quad \sigma_2 = \sqrt{1} = 1.$

EIGENVECTORS OF A^TA: $A^TA - 6I = \begin{pmatrix} -4 & -2 \\ 2 & -1 \end{pmatrix}$ NORMALIZED EVECTOR $\underline{v}_1 = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

$A^TA - I = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ " " $\underline{v}_2 = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

TO FIND $\underline{u}_1$, NORMALIZE $A\underline{v}_1$, OR SAME, NORMALIZE $A\begin{pmatrix} 1 \\ 2 \end{pmatrix}$:

$A\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} \Rightarrow \underline{u}_1 = \frac{1}{\sqrt{30}}\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$

$A\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \Rightarrow \underline{u}_2 = \frac{1}{\sqrt{5}}\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$.

TO FIND $\underline{u}_3$, FIND NORMALIZED VECTOR IN $N(A^T)$

ELIMINATE:

$A^T = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$ SPECIAL SOLN. $\begin{pmatrix} -1/2 \\ 1/2 \\ 1 \end{pmatrix}$ EASIER TO NORMALIZE $\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$

$\Rightarrow \underline{u}_3 = \frac{1}{\sqrt{6}}\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$.

So.

$$A = \underbrace{\begin{pmatrix} \frac{5}{\sqrt{30}} & 0 & -1/\sqrt{6} \\ 1/\sqrt{30} & 2/\sqrt{5} & 1/\sqrt{6} \\ 2/\sqrt{30} & -1/\sqrt{5} & 2/\sqrt{6} \end{pmatrix}}_{U} \underbrace{\begin{pmatrix} \sqrt{6} & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}}_{\Sigma} \underbrace{\begin{pmatrix} 1/\sqrt{5} & 2/\sqrt{5} \\ 2/\sqrt{5} & -1/\sqrt{5} \end{pmatrix}}_{V^T\ (=V)}.$$

IS THE DISCRETE FOURIER TRANSFORM

$$\begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \frac{F_4^H}{4} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

(THE DEFINITION OF HERMITIAN TRANSPOSE)

$$F_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & w & w^2 & w^3 \\ 1 & w^2 & w^4 & w^6 \\ 1 & w^3 & w^6 & w^9 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}, \quad F_4^H = \overline{F_4^T} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix}$$

$$w = e^{2\pi i/4} = e^{i\pi/2} = i$$

$$(F_4 = F_4^T)$$

SO

$$\begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ i \\ 1 \\ -i \end{pmatrix}.$$

$$|c_1| = |c_2| = |c_3| = 1. \quad |c_0| = 3.$$

c_0 IS THE LARGEST FOURIER COEFFICIENT.

TRUE/FALSE QUESTIONS.

a : FALSE, SWAPPING ROWS FLIPS SIGN

b : FALSE, $\left| \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right| \neq 0$.

c : TRUE : $AA^{-1} = I \Rightarrow |A||A^{-1}| = 1$.

d : TRUE : ONE TERM IN THE BIG FORMULA.

e : FALSE : $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

f : TRUE : EIGENVECTORS FROM DISTINCT EIGENVALUES ARE INDEPENDENT.

g : FALSE : $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

h : TRUE : HAS ORTHONORMAL BASIS OF EIGENVECTORS.

i : FALSE : IF $A = S\Lambda S^{-1}$ WITH $S^{-1} = S^T$ THEN $A = S\Lambda S^T$ IS SYMMETRIC. So $\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$ IS A COUNTER EXAMPLE

j : FALSE $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is A COUNTEREXAMPLE ($\lambda = 1$ ONLY EIGENVALUE)

k : TRUE : $|(A - \lambda I)^T| = |A - \lambda I|$, SO THESE POLYNOMIALS HAVE THE SAME ROOTS.

l : FALSE : $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ evector $\underset{(\lambda=1)}{}$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $A^T = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ evector $(\lambda = 1)$. $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

m : TRUE : $A\underline{v}_1 = \lambda_1 \underline{v}_1$ $A\underline{v}_2 = \lambda_2 \underline{v}_2$, $\lambda_1 \neq \lambda_2$, then

$$\lambda_1(\underline{v}_1 \cdot \underline{v}_2) = \underline{v}_2^T A \underline{v}_1 = (A\underline{v}_2)^T \underline{v}_1 = \underline{v}_1 \cdot A\underline{v}_2 = \lambda_2 \underline{v}_1 \cdot \underline{v}_2.$$

$$\Rightarrow (\lambda_2 - \lambda_1)\underline{v}_1 \cdot \underline{v}_2 = 0 \Rightarrow \underline{v}_1 \cdot \underline{v}_2 = 0.$$

n : FALSE: $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

o : TRUE : A_1, A_2 pos. def. then $\underline{x}^T(A_1 + A_2)\underline{x} = \underline{x}^T A_1 \underline{x} + \underline{x}^T A_2 \underline{x} > 0$ FOR $\underline{x} \neq 0$.

$\Rightarrow A_1 + A_2$ POS. DEF.

p : TRUE : ALGORITHM ALWAYS WORKS.

q : FALSE : EG. $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \to \begin{pmatrix} \boxed{2} & 1 \\ 0 & \boxed{1/2} \end{pmatrix}$ BUT ITS EIGENVALUES ARE NOT 2 AND $1/2$

r : FALSE : IDENTITY.

s : FALSE : $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ has eigenvalues $\lambda_1 + \lambda_2 = \text{trace} = 3$

$\lambda_1 \lambda_2 = 1$.

So one of them $\neq 1$.

t : TRUE : $N(A)$ has a nonzero vector.

u : TRUE: LEARNED IN LECTURE (GOOD ENOUGH IN THIS INSTANCE).

TRUE/FALSE QUESTIONS CONTINUED:

v TRUE FOLLOWS FROM THE BIG FORMULA FOR $|A-\lambda I|$, FOR EXAMPLE.
(THERE IS ONLY ONE TERM)

X TRUE FOLLOWS FROM PROPERTY OF DETERMINANT (ROW EXCHANGE FLIPS SIGN).

W FALSE, THE SINGULAR VALUES ARE NONNEGATIVE REAL NUMBERS, BUT Λ CAN HAVE NEGATIVE ENTRIES (EIGENVALUES OF A) E.G. $A = \begin{pmatrix} -2 & 1 \\ 0 & 1 \end{pmatrix}$ HAS A NEGATIVE EVALUE (-2)

y. TRUE IN THE FOLLOWING SENSE: IF $A\underline{v} = \lambda \underline{v}$ $A^2 \underline{v} = \lambda^2 \underline{v}$

AND ALL EIGENVALUES OF A^2 ARE OBTAINED BY SQUARING AN EIGENVALUE OF A (THIS FOLLOWS FROM JORDAN NORMAL FORM).

ON THE OTHER HAND, THEIR MULTIPLICITIES CAN BE DIFFERENT. (CONSIDER $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$).

z. FALSE, CONSIDER $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, EACH HAS ONLY ZERO EIGENVALUE

$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, BUT THE SUM HAS NONZERO DETERMINANT (HENCE ALL EIGENVALUES $\neq 0$).