AMTH/MATH 222 Practice Midterm Questions

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Information about the midterm

The midterm covers everything up to and including the Gram-Schmidt process and the QR decomposition. So the midterm will cover chapters 1,2,3 and 4 of the textbook.

The midterm is going to take in class, and therefore take **50 minutes.**

There are going to be **3** questions on the midterm.

There will be two different types of question.

There will be **2** questions involving calculations.

There will **also** be a true or false question with **multiple parts**.

Questions involving a calculation (show your working).

1. Find the complete solution to the system of linear equations

$$\begin{pmatrix} 1 & 3 & 1 & 2 \\ 2 & 7 & 4 & 8 \\ 1 & 4 & 3 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}.$$

2. If it exists, find the inverse of the matrix

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

3. Find a basis for the 2D plane in $\mathbb{R}^4$ given by the equations

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 0 \\ x_2 + x_3 &= 0. \end{aligned}$$

4. Put the matrix

$$\begin{pmatrix} 1 & 1 & 2 & 4 \\ 1 & 2 & 2 & 5 \\ 1 & 3 & 2 & 6 \end{pmatrix}$$

into reduced row echelon form.

5. Find an orthonormal basis for the column space of the matrix

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & 3 & 6 \end{pmatrix}.$$

Then write $A = QR$.

6. Find the least squares solution to $A\underline{x} = \underline{b}$ where A is the matrix of Question 5. Do this 2 ways: with and without the QR decomposition of A.

7. Find the line of best fit to the points $(0,0), (1,8), (3,8), (4,20)$.

8. Find the matrix giving the projection onto the column space of A where

$$A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

9. Find the rank of the matrix

$$A = \begin{pmatrix} 0 & 0 & 1 & -2 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 1 & 0 & 0 & -3 \end{pmatrix}.$$

10. Find a permutation P, lower triangular matrix L and upper triangular U such that

$$PA = LU$$

where

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 7 & 9 \end{pmatrix}.$$

True or False questions

For each question, unless otherwise stated, A is an $m \times n$ matrix. You should be able to give a short explanation for your answer.

a. There is a 4×9 matrix with the same dimension column space and null space.

b. If A has independent rows, then $N(A) = \{\underline{0}\}$.

c. If A has independent columns, then $C(A) = \mathbb{R}^m$.

d. $C(A)$ and $N(A)$ are orthogonal complements in $\mathbb{R}^n$.

e. $C(A^T)$ is in $\mathbb{R}^n$.

f. There exist 101 independent vectors in $\mathbb{R}^{100}$.

g. Any 100 independent vectors in $\mathbb{R}^{100}$ span $\mathbb{R}^{100}$.

h. If 100 vectors space $\mathbb{R}^{100}$, then they are independent.

i. Any 3 nonzero vectors in $\mathbb{R}^3$ span a vector space of dimension 3.

j. The column space of A is the same as its row reduced echelon form R.

k. We can always write $A = LU$ where L is lower triangular and U is upper triangular.

l. The pivot columns of A are a basis for the column space of A.

m. The rank of A is n. Then $A\underline{x} = \underline{b}$ has a solution for any $\underline{b}$.

n. A row of A can be in the null space of A.

o. V is a subspace of $\mathbb{R}^n$. P is the orthogonal projection onto V. Then $P\underline{v}$ is always orthogonal to $\underline{v}$.

p. There is a pair of orthogonal vector spaces, each of dimension 4, inside $\mathbb{R}^8$.

q. It is always possible to find at least one least squares solution to a system of linear equations.

r. If $\underline{b}$ is in the column space of A, then $A\underline{x} = \underline{b}$ has a solution.

s. Every vector subspace V of R^n has a unique orthogonal complement.

t. A permutation is always an orthogonal matrix.

u. A permutation is always a symmetric matrix.

v. $A^T A$ is a symmetric matrix.

w. If A is square $(m = n)$ and $BA = I$, then $AB = I$.

x. An orthonormal set of vectors is linearly independent.

y. If A is invertible, then the columns of A are linearly independent.

z. If $AB = I$, then $BA = I$.