Erratum to: Quantum affine Gelfand-Tsetlin bases and quantum toroidal algebra via K-theory of affine Laumon spaces (Sel. Math. New Ser. 16 (2010): 173–200)

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Following the work of Feigin-Finkelberg-Negut-Rybnikov, we construct an action of the quantum loop algebra $\mathbf{U}_v(\mathbf{L}\mathfrak{sl}_n)$ on the sum of localized $\widetilde{T}\times\mathbb{C}^*$–equivariant K-groups of Laumon spaces $\mathfrak{Q}_{\underline{d}}$ (Theorem 2.12, p. 179). We also construct an action of the quantum toroidal algebra $\ddot{\mathrm{U}}_v(\widehat{\mathfrak{sl}}_n)$ on the sum of localized $\widetilde{T}\times\mathbb{C}^*\times\mathbb{C}^*$–equivariant K-groups of the moduli space of parabolic sheaves $\mathcal{P}_{\underline{d}}$ (*affine analogues of Laumon spaces*) (Theorem 4.13, p. 196).

Corrections

(0) In the definition of Laumon spaces (*loc. cit. p. 174, line -5*), the following should be added right before $\mathfrak{Q}_{\underline{d}}\subset\mathcal{Q}_{\underline{d}}$:
"We consider the following locally closed subvariety $\mathfrak{Q}_{\underline{d}}\subset\mathcal{Q}_{\underline{d}}$ (quasiflags based at $\infty\in\mathbf{C}$)...".

(1) In the very end of Section 2.2 (*loc. cit. p. 175, line 12*), the following should be added:
"*Notation:* Given a collection $\underline{\widetilde{d}}$ as above, we will denote by $\underline{\widetilde{d}}+\delta_{i,j}$ the collection $\underline{\widetilde{d}}'$, such that $\underline{\widetilde{d}}'_{i,j}=\underline{\widetilde{d}}_{i,j}+1$, while $\underline{\widetilde{d}}'_{p,q}=\underline{\widetilde{d}}_{p,q}$ for $(p,q)\neq(i,j)$ (in all our cases it will satisfy the required conditions, though in general as defined it might not)."

Similar comment should be added in the Section 4.4 (*loc. cit. p. 193, line 16*):
"*Notation:* Given a collection $\underline{\widetilde{d}}$ as above we will denote by $\underline{\widetilde{d}}+\delta_{i,j}$ the collection $\underline{\widetilde{d}}'$, such that $\underline{\widetilde{d}}'_{i+ns,j+ns}=\underline{\widetilde{d}}_{i,j}+1$ ($\forall s\in\mathbb{Z}$), while $\underline{\widetilde{d}}'_{p.q}=\underline{\widetilde{d}}_{p,q}$ for all other (p,q)."

(2) In section 2.11 series $\mathbf{b}_m(z)$ (*loc. cit. p. 178–179, lines -4 – 3*) and $\mathbf{b}_{mi}(z)$ (*loc. cit. p. 180, lines -8 – -5*) were introduced and played an important role in the construction of $\psi_i^{\pm}(z)$ operators, Theorem 2.12 (*loc. cit. p. 179, lines 14–21*). One should change their definition as follows. Let $\pi:\mathfrak{Q}_{\underline{d}}\times(\mathbf{C}\backslash\{\infty\})\to\mathfrak{Q}_{\underline{d}}$ denote the standard projection. Then we set:

$$\mathbf{b}_i(z):=\Lambda^{\bullet}_{-1/z}(\pi_*(\underline{\mathcal{W}}_i\mid_{\mathbf{C}\backslash\{\infty\}}))=1+\sum_{j\geq1}\Lambda^j(\pi_*(\underline{\mathcal{W}}_i\mid_{\mathbf{C}\backslash\{\infty\}}))(-z^{-1})^j:\ M_{\underline{d}}\to M_{\underline{d}}[[z^{-1}]],$$

$$\mathbf{b}_{mi}(z):=\Lambda^{\bullet}_{-1/z}(\pi_*(\underline{\mathcal{W}}_{mi}\mid_{\mathbf{C}\backslash\{\infty\}})):\ M_{\underline{d}}\to M_{\underline{d}}[[z^{-1}]].$$

Similarly we define operators $\mathbf{b}_{mi}(z)$ in section 4.11 (*loc. cit. p. 195, lines 8–11*).

(3) Operators $\mathfrak{f}_i$ and $f_{i,r}$ should be multiplied by v throughout paper. In particular:

- In formula (8) (*loc. cit. p. 176, line -5*) $\mathfrak{f}_i = -t_i^{-1} v^{d_i - d_{i-1}+i} \mathbf{q}_*(L_i \otimes \mathbf{p}^*)$.
- In formula (16) (*loc. cit. p. 179, line 10*) $f_{k,r} = -t_k^{-1} v^{d_k - d_{k-1}+k} \mathbf{q}_*(L_k \otimes (L'_k)^{\otimes r} \otimes \mathbf{p}^*)$.

In the definition of $\mathfrak{f}_i, f_{k,r}, e_{k,r}$ in the parabolic setting we should also change the powers of u and indices of line bundles L_i. In particular:

- In formula (38) (*loc. cit. p. 194, line 11*) $\mathfrak{f}_i = -t_i^{-1} u^{-\delta_{i,n}} v^{d_i - d_{i-1}+i} \mathbf{q}_*(L_{i-n} \otimes \mathbf{p}^*)$.
- In formula (47) (*loc. cit. p. 195, line -1*) $f_{k,r}$ is given by
$f_{k,r} = -t_k^{-1} u^{-\delta_{i,n}} v^{d_k - d_{k-1}+k} \mathbf{q}_*(L_{k-n} \otimes (L'_{k-n})^{\otimes r} \otimes \mathbf{p}^*)$.
- In formula (46) (*loc. cit. p. 195, line -2*) $e_{k,r} = t_{k+1}^{-1} v^{d_{k+1} - d_k + 1 - k} \mathbf{p}_*((L'_{k-n})^{\otimes r} \otimes \mathbf{q}^*)$.

(4) In the proof of relation (23) (*loc. cit. p. 187, line -4*) there is a typo in the definition of r_k and p_k. It should be corrected in the following way:
$p_k := s_{i-1,k} = t_k^2 v^{-2d_{i-1,k}}, \; r_k := s_{i+1,k} = t_k^2 v^{-2d_{i+1,k}}$.

(5) Also in the proof of (23) (*loc. cit. pp. 187–189*), all formulas (*p. 187, line -2; p. 188, line 3; p. 188, line -5; p. 189, line 3*) for $\varphi^+_{i,a}$ and $\varphi^+_{i,a+1}$ should be multiplied by an additional common factor, which doesn't affect the equality:
$-t_{i+1}^{-1} t_i^{-1} v^{-1} (v^2-1)^{-1} v^{d_{i+1} - d_{i-1}}$.

(6) In the description of fixed points in formula (30) (*loc. cit. p. 191, lines -2 – -1*):
all the inclusions $\subset$ and $\widetilde{\subset}$ should be reversed.

(7) Formula for the ideal J_λ from Section 4.1 (*loc. cit. p. 191, line -7*) should read as follows:
$J_\lambda = \mathbb{C}[y,z] \cdot (\mathbb{C}y^0 z^{\lambda_0} \oplus \mathbb{C}y^1 z^{\lambda_1} \oplus \cdots)$.

(8) In the definition of operators $\mathfrak{k}_i$ (*loc. cit. Section 4.8, p. 194, line 9*) and generating series $\psi_i^\pm(z)$ (*loc. cit. Section 4.11, p. 195, lines 16–17*) the power of u should be corrected in the following way:

- Formula (36) (*loc. cit. p. 194, line 9*) should read as $\mathfrak{k}_i = t_{i+1}^{-1} t_i u^{-\delta_{i,n}} v^{-2d_i + d_{i-1} + d_{i+1} - 1}$.
- Formula (42) (*loc. cit. p. 195, lines 16–17*) should define $\psi_i^\pm(z)$ as:
$\frac{t_i}{t_{i+1}} u^{-\delta_{i,n}} v^{d_{i+1} - 2d_i + d_{i-1} - 1} \left(\mathbf{b}_{m,i-n}(zv^{-i-2})^{-1} \mathbf{b}_{m,i-n}(zv^{-i})^{-1} \mathbf{b}_{m,i-n-1}(zv^{-i}) \mathbf{b}_{m,i-n+1}(zv^{-i-2})\right)^\pm$.

(9) In the renormalization of vectors in (50) (*loc. cit. p. 198, line 8*) as well as in the formulas for $e_{i,r\langle \underline{\widetilde{d}'}, \underline{\widetilde{d}} \rangle}, f_{i,r\langle \underline{\widetilde{d}'}, \underline{\widetilde{d}} \rangle}$ (*loc. cit. p. 198, lines -2 – -1*) the following change is required:
all products of the form $\prod_{\dots} w$ should be corrected to $\prod_{\dots} (1-w)$.

(10) In the definition of $p_{i,j}$ from Proposition 4.15 (*loc. cit. p. 197, line 2*) the formula should read as follows:
$p_{i,j} := t^2_{j \pmod n} v^{-2d_{ij}} u^{-2\lfloor \frac{-j+n}{n} \rfloor} = t^2_{j \pmod n} v^{-2d_{ij}} u^{2\lceil \frac{j-n}{n} \rceil}$.

(11) In the proof of the main Theorem 4.13 (*loc. cit. p. 197, lines 17–23*), the following argument should be added in the beginning:
"For any $k \in \mathbb{Z}$ we define $x_k^\pm(z), \psi_k^\pm(z)$ by the same formulas (42–47) with $\delta_{k,n}$ being changed to $\delta_{k \pmod n), 0}$.

First, because of the above remark and our computational proof of Theorem 2.12, relations (9–14) still hold. Indeed, relations (12–14) are verified along the same lines with just $p_{i,j}$ instead of $s_{i,j}$. Similarly with (9–10). The only nontrivial equality is $\psi^+_{i,0} - \psi^-_{i,0} = \chi_{i,0}$, where $\chi_{i,0}$ is defined in the same way with p_{ij}'s instead of s_{ij}'s. However, it is a statement of Theorem 4.9.[1] The relation (11) follows."

Formulas for $\hat{\psi}^{\pm}_n(z),\ \hat{x}^{\pm}_n(z)$ (*loc. cit. p. 197, lines 19–21*) therein should read as follows:

$$\hat{\psi}^{\pm}_n(z) = \psi^{\pm}_0(z),\ \hat{x}^+_n(z) = v^{-n}x^+_0(z),\ \hat{x}^-_n(z) = v^n u^2 x^-_0(z).$$

(12) Some verifications should be added to the proof of Theorem 4.19 (p. 199, line 14):

"Both verifications are straightforward and we will sketch only those for $e_{i,r}$ operators. Under the above specialization, for $j = nj_0 + j_1$ $(j_0 \in \mathbb{Z}, 1 \le j_1 \le n)$, we get

$$p_{i,j} = v^{2\widetilde{\mu}_{j_1} - 2j_1 + 2 - 2d_{i,j} - 2j_0(K+n)} = v^{2(\widetilde{\mu}_j - j - d_{i,j} + 1)}.$$

(i) We need to show $\widetilde{\mu}_j - j - d_{i,j} \ne \widetilde{\mu}_k - k - d_{i,k} - 1,\ \forall k \le i$, for $\underline{\widetilde{d}} \in D(\mu)$ such that $\underline{\widetilde{d}} - \delta^j_i \in D$.
∘ If $j \le k \le i$, then $d_{i,j} - \widetilde{\mu}_j \le d_{i+k-j,k} - \widetilde{\mu}_k \le d_{i,k} - \widetilde{\mu}_k$ and $j < k+1$, implying the result.
∘ If $k < j \le i$, then $d_{i,k} - \widetilde{\mu}_k \le d_{i+j-k,j} - \widetilde{\mu}_j \le d_{i,j} - \widetilde{\mu}_j$ and $k+1 \le j$. This implies $d_{i,k} - \widetilde{\mu}_k + k + 1 \le d_{i,j} - \widetilde{\mu}_j + j$. However, if the equality happens above, then we have $j = k+1$ and $d_{i+j-k,j} = d_{i,j}$, that is $d_{i+1,j} = d_{i,j}$. But this contradicts our assumption $\underline{\widetilde{d}} - \delta^j_i \in D$.

(ii) We need to prove an existence of $k \le i-1$ satisfying $\widetilde{\mu}_j - j - d_{i,j} = \widetilde{\mu}_k - k - d_{i-1,k} - 1$ for $\underline{\widetilde{d}} \in D(\mu)$, such that $\underline{\widetilde{d}} - \delta^j_i \in D \backslash D(\mu)$.

Recalling the definition of $D(\mu)$, the latter condition on $\underline{\widetilde{d}}$ guarantees $d_{i-l,j-l} - \widetilde{\mu}_{j-l} = d_{i,j} - \widetilde{\mu}_j$ for some $l \ge 1$ and so $d_{i-1,j-1} - \widetilde{\mu}_{j-1} = d_{i,j} - \widetilde{\mu}_j$. Thus, picking $k := j-1$ works."

(13) The following references have been published since then (*loc. cit. p. 199, lines -4 – -1*):

• B. Feigin, M. Finkelberg, I. Frenkel, L. Rybnikov, *Gelfand-Tsetlin algebras and cohomology rings of Laumon spaces*, Sel. Math. New Ser. **17** (2011), 337–361.

• B. Feigin, M. Finkelberg, A. Negut, L. Rybnikov, *Yangians and cohomology rings of Laumon spaces*, Sel. Math. New Ser. **17** (2011), 573–607.

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[1] Actually, it reduces to the equality from the proof of Proposition 2.21, reference #2 of *loc. cit.*. The point why $u^{-\delta_{i,n}}$ appears now is that $\prod_{j \le i+1} p_{i+1,j} \prod_{j \le i} p^{-1}_{i,j} = t^2_{i+1} u^{2\lceil \frac{i+1-n}{n} \rceil} v^{2d_i - 2d_{i+1}}$, while for $s_{i,j}$ we had the same equality without $u^{2\lceil \frac{i+1-n}{n} \rceil}$.