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Following the work of Feigin-Finkelberg-Negut-Rybnikov, we construct an action of the quantum loop algebra $U_v(L sl_n)$ on the sum of localized $\mathcal{T} \times \mathbb{C}^*$–equivariant $K$-groups of Laumon spaces (Theorem 2.12, p. 179). We also construct an action of the quantum toroidal algebra $\tilde{U}_v(b sl_n)$ on the sum of localized $\mathcal{T} \times \mathbb{C}^* \times \mathbb{C}^*$–equivariant $K$-groups of the moduli space of parabolic sheaves $P_d$ (affine analogues of Laumon spaces) (Theorem 4.13, p. 196).

Corrections

(0) In the definition of Laumon spaces (loc. cit. p. 174, line -5), the following should be added right before $Q_d \subset Q_d$:

“We consider the following locally closed subvariety $Q_d \subset Q_d$ (quasiflags based at $\infty \in \mathbb{C}$)...”.

(1) In the very end of Section 2.2 (loc. cit. p. 175, line 12), the following should be added:

“Notation: Given a collection $\tilde{d}$ as above, we will denote by $\tilde{d} + \delta_{i,j}$ the collection $\tilde{d}'$, such that $\tilde{d}'_{i,j} = \tilde{d}_{i,j} + 1$, while $\tilde{d}'_{p,q} = \tilde{d}_{p,q}$ for $(p,q) \neq (i,j)$ (in all our cases it will satisfy the required conditions, though in general as defined it might not).”

Similar comment should be added in the Section 4.4 (loc. cit. p. 193, line 16):

“Notation: Given a collection $\tilde{d}$ as above we will denote by $\tilde{d} + \delta_{i,j}$ the collection $\tilde{d}'$, such that $\tilde{d}_{i,j} + \delta_{s,j} + \delta_{s,\infty} = \tilde{d}'_{i,j} + 1$ ($\forall s \in \mathbb{Z}$), while $\tilde{d}_{p,q}' = \tilde{d}_{p,q}$ for all other $(p,q)$.”

(2) In section 2.11 series $b_m(z)$ (loc. cit. p. 178–179, lines -4 – 3) and $b_{mi}(z)$ (loc. cit. p. 180, lines -8 – -5) were introduced and played an important role in the construction of $\psi_\pm(z)$ operators, Theorem 2.12 (loc. cit. p. 179, lines 14–21). One should change their definition as follows. Let $\pi : \Omega_d \times (\mathbb{C} \setminus \{\infty\}) \to \Omega_d$ denote the standard projection. Then we set:

$$b_i(z) := \Lambda^{+1/z} \pi_* (W |_{\mathbb{C} \setminus \{\infty\}}) = 1 + \sum_{j \geq 1} \Lambda^j (\pi_* (W |_{\mathbb{C} \setminus \{\infty\}})(-z^{-1})^j : M_d \to M_d[[z^{-1}]],$$

$$b_{mi}(z) := \Lambda^{+1/z} \pi_* (W_{mi} |_{\mathbb{C} \setminus \{\infty\}}) : M_d \to M_d[[z^{-1}]].$$
Similarly we define operators $b_{n\ell}(z)$ in section 4.11 (loc. cit. p. 195, lines 8–11).

(3) Operators $f_i$ and $f_{i,r}$ should be multiplied by $v$ throughout paper. In particular:
- In formula (8) (loc. cit. p. 176, line -5) $f_i = -t_i^{-1}v^{d_i-d_i-1+i}q_i(L_i \otimes p^*)$.
- In formula (16) (loc. cit. p. 179, line 10) $f_{k,r} = -t_k^{-1}v^{d_k-d_k-1+k}q_k(L_k \otimes (L_k')^{\otimes r} \otimes p^*)$.

In the definition of $f_i, f_{k,r}, e_{k,r}$ in the parabolic setting we should also change the powers of $u$ and indices of line bundles $L_i$. In particular:
- In formula (38) (loc. cit. p. 194, line 11) $f_i = -t_i^{-1}u^{-\delta_i,n}v^{d_i-d_i-1+i}q_i(L_i-n \otimes p^*)$.
- In formula (47) (loc. cit. p. 195, line -1) $f_{k,r}$ is given by $f_{k,r} = -t_k^{-1}u^{-\delta_k,n}v^{d_k-d_k-1+k}q_k(L_k-n \otimes (L_k-n')^{\otimes r} \otimes p^*)$.
- In formula (46) (loc. cit. p. 195, line -2) $e_{k,r} = t_k^{1+t}u^{d_k+1-d_k+1+k}p_s((L_k'-k-n')^{\otimes r} \otimes q^*)$.

(4) In the proof of relation (23) (loc. cit. p. 187, line -4) there is a typo in the definition of $r_k$ and $p_k$. It should be corrected in the following way:

$$p_k := s_{i+1,k} = t_k^2v^{-2d_i+1,k}, \quad r_k := s_{i+1,k} = t_k^2v^{-2d_i+1,k}.$$  

(5) Also in the proof of (23) (loc. cit. pp. 187–189), all formulas (p. 187, line -2; p. 188, line 3; p. 188, line -5; p. 189, line 3) for $\varphi_{i,a}$ and $\varphi_{i,a+1}$ should be multiplied by an additional common factor, which does not affect the equality:

$$-t_i^{-1}v^{1-(-u^2-1)}-t_i^{-1}v^{-1+d_i-1}.$$  

(6) In the description of fixed points in formula (30) (loc. cit. p. 191, lines -2 -1):
- all the inclusions $\subset$ and $\supset$ should be reversed.

(7) Formula for the ideal $J_\lambda$ from Section 4.1 (loc. cit. p. 191, line -7) should read as follows:

$$J_\lambda = C[y, z] \cdot (C[y^0z^{\lambda_0} \oplus Cy^1z^{\lambda_1} \oplus \cdots]).$$  

(8) In the definition of operators $t_i$ (loc. cit. Section 4.8, p. 194, line 9) and generating series $\psi_i^\pm(z)$ (loc. cit. Section 4.11, p. 195, lines 16–17) the power of $u$ should be corrected in the following way:
- Formula (36) (loc. cit. p. 194, line 9) should read as $t_i = t_i^{1-n}u^{-\delta_i,n}v^{-2d_i+d_i+1}i$.  
- Formula (42) (loc. cit. p. 195, lines 16–17) should define $\psi_i^\pm(z)$ as:

$$t_i^{-1}u^{-\delta_i,n}v^{-2d_i+d_i+1}i \left( b_{m,i-n}(zv^{-1})^{-1}b_{m,i-n-1}(zv^{-2}) \right)^\pm.$$  

(9) In the renormalization of vectors in (50) (loc. cit. p. 195, line 8) as well as in the formulas for $e_{i,r}(\overrightarrow{d}), f_{i,r}(\overrightarrow{d})$ (loc. cit. p. 198, lines -2 -1) the following change is required:
- all products of the form $\prod_\infty w$ should be corrected to $\prod_\infty (1-w)$.

(10) In the definition of $p_i,j$ from Proposition 4.15 (loc. cit. p. 197, line 2) the formula should read as follows:

$$p_{i,j} := t_j^2 (\mod n) v^{-2d_{i,j}}(\frac{-2d_{i,j}}{n}) = t_j^2 (\mod n) v^{-2d_{i,j}}u^{2[\frac{-2d_{i,j}}{n}]}.$$  

(11) In the proof of the main Theorem 4.13 (loc. cit. p. 197, lines 17–23), the following argument should be added in the beginning:

“For any $k \in \mathbb{Z}$ we define $x_k^\pm(z), \psi_k^\pm(z)$ by the same formulas (42–47) with $\delta_{k,n}$ being changed to $\delta_{k,n}$.

First, because of the above remark and our computational proof of Theorem 2.12, relations (9–14) still hold. Indeed, relations (12–14) are verified along the same lines with just $p_{i,j}$ instead
of $s_{i,j}$. Similarly with (9–10). The only nontrivial equality is $\psi_{i,0}^+ - \psi_{i,0}^- = \chi_{i,0}$, where $\chi_{i,0}$ is defined in the same way with $p_{i,j}$’s instead of $s_{i,j}$’s. However, it is a statement of Theorem 4.9.\footnote{Actually, it reduces to the equality from the proof of Proposition 2.21, reference #2 of loc. cit. The point why $u^{-d_{i,s}}$ appears now is that $\prod_{j \leq i} p_{i,j+1}^{(i,j+1)} \prod_{j \geq i} p_{i,j}^{-1} = 1^{2d_{i,s}} u^{2d_{i,s} \left( \frac{K+1}{n} \right) \frac{K+2}{n} - 2d_{i,s} - 2d_{i,s}}$, for $s_{i,j}$ we had the same equality without $u^{2d_{i,s}}$.}

The relation (11) follows.”

Formulas for $\hat{\psi}_n^+(z)$, $\hat{x}_n^+(z)$ (loc. cit. p. 197, lines 19–21) therein should read as follows:

$$\hat{\psi}_n^+(z) = \psi_0^+(z), \quad \hat{x}_n^+(z) = v^{-n} x_0^+(z), \quad \hat{x}_n^-(z) = v^n u^2 x_0^-(z).$$

(12) Some verifications should be added to the proof of Theorem 4.19 (p. 199, line 14):

“Both verifications are straightforward and we will sketch only those for $e_{i,r}$ operators. Under the above specialization, for $j = nj_0 + j_1$ ($j_0 \in \mathbb{Z}, 1 \leq j_1 \leq n$), we get

$$p_{i,j} = v^{2d_{i,j} - 2j_1 + 2 - 2d_{i,j} - 2j_0(K+n)} = v^{2d_{i,j} - d_{i,j} + 1}.$$

(i) We need to show $\tilde{p}_{ij} - j - d_{i,j} \neq \tilde{\mu}_k - k - d_{i,k} - 1$, $\forall k \leq i$, for $\tilde{d} \in D(\mu)$ such that $\tilde{d} - \delta_i \in D$.

- If $j \leq k \leq i$, then $d_{i,j} - \tilde{p}_j \leq d_{i+k-j,k} - \tilde{\mu}_k \leq d_{i,k} - \tilde{\mu}_k \leq j + k < j + 1$, implying the result.

- If $k < j \leq i$, then $d_{i,k} - \tilde{\mu}_k \leq d_{i+k-j,k} - \tilde{\mu}_k \leq d_{i,j} - \tilde{\mu}_j + k + 1 \leq d_{i,j} - \tilde{\mu}_j + j$. However, if the equality happens above, then we have $j = k + 1$ and $d_{i,j} = d_{i+1,j} = d_{i,j}$. But this contradicts our assumption $\tilde{d} - \delta_i \in D$.

(ii) We need to prove an existence of $k \leq i - 1$ satisfying $\tilde{p}_{ij} - j - d_{i,j} = \tilde{\mu}_k - k - d_{i-1,k} - 1$ for $\tilde{d} \in D(\mu)$, such that $\tilde{d} - \delta_i \in D(\mu)$.

Recalling the definition of $D(\mu)$, the latter condition on $\tilde{d}$ guarantees $d_{i-1,j-1} - \tilde{\mu}_{j-1} = d_{i,j} - \tilde{\mu}_j$ for some $l \geq 1$ and so $d_{i-1,j-1} - \tilde{\mu}_{j-1} = d_{i,j} - \tilde{\mu}_j$. Thus, picking $k := j - 1$ works.”

(13) The following references have been published since then (loc. cit. p. 199, lines -4 – -1):


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