

I retired from administration and teaching at the end of 2003, but remained active in research.

Per *Ecclesiastes*, “of making many books there is no end.” The notes from Math 301–305 (Analysis) were turned into a textbook – my second undergraduate analysis textbook. (The first, published in 1973, tried to use the theory of distributions to make everything easier. It has seen little use, even by me.) The newer text, *Analysis, an Introduction*, was published by Cambridge in 2004. It is much more classical in approach, and gets to some fun topics including Euler’s product formula for  $\sin x$ , the Banach–Tarski paradox, and the Heisenberg uncertainty principle. It has been used for Math 301-305 at Yale but, judging from the yearly sales, sees little use elsewhere. For anyone who might be interested, the table of contents and a list of corrections are appended.

Through various research projects I was drawn, reluctantly, into a passing acquaintance with special functions (the ones that you don’t learn about in calculus but that have names nonetheless). Some random events led to co-authoring a book on the subject, with Roderick Wong of the City University of Hong Kong, who is an actual expert in the subject. Cambridge University Press took it on as well: *Special Functions, a Graduate Text*. See below end for a few corrections. A second, enlarged, edition, is retitled as *Special Functions and Orthogonal Polynomials*; its table of contents is given below.

Several post-retirement stints teaching the advanced complex analysis course showed me that the wonderful thing about the course is that there are too many great topics to cover in one iteration, so that the instructor and the class get to make some choices. So of course one needs a book to cover a large selection of topics – and Wong and I have produced one, *Exporations in Complex Functions*, published by Springer in 2020. Again, the table of contents is given below.

## **Tables of Contents, in reverse chronological order**

### **Explorations in Complex Functions – Contents:**

1. Basics
2. Linear fractional transformations
3. Hyperbolic geometry
4. Harmonic functions
5. Conformal maps and the Riemann mapping theorem
6. The Schwarzian derivative
7. Riemann surfaces and algebraic curve
8. Entire functions
9. Value distribution theory
10. The gamma and beta functions
11. The Riemann zeta function
12.  $L$ -functions and primes
13. The Riemann hypothesis
14. Elliptic functions and theta functions
15. Jacobi elliptic functions

16. Weierstrass elliptic functions
17. Automorphic functions and Picard's theorem
18. Integral transforms
19. Theorems of Phragmén–Lindelöf and Paley–Wiener
20. Theorems of Wiener and Lévy; the Wiener–Hopf method
21. Tauberian theorems
22. Asymptotics and the method of steepest descent
23. Complex interpolation and the Riesz–Thorin theorem

**Special Functions and Orthogonal Polynomials – Contents:**

1. Introduction
2. Gamma, beta, zeta
3. Second-order differential equations
4. Orthogonal polynomials on an interval
5. The classical orthogonal polynomials
6. Semi-classical orthogonal polynomials
7. Asymptotics of orthogonal polynomials: two methods
8. Confluent hypergeometric functions
9. Cylinder functions
10. Hypergeometric functions
11. Spherical functions
12. Generalized hypergeometric functions;  $G$ -functions
13. Asymptotics
14. Elliptic functions
15. Painlevé transcendents
- Appendix A. Complex analysis
- Appendix B. Fourier analysis

**Analysis, an Introduction – Contents:**

1. Introduction
2. The Real and Complex Numbers
3. Real and Complex Sequences
4. Series
5. Power Series
6. Metric Spaces
7. Continuous Functions
8. Calculus
9. Some Special Functions
10. Lebesgue Measure on the Line
11. Lebesgue Integration on the Line
12. Function Spaces
13. Fourier Series
14. Applications of Fourier Series
15. Ordinary Differential Equations

I am indebted to Mary Pugh, Gerard Misiolek, and José Rodrigo for most of the following corrections. And I am especially indebted to Eric Belsley, whose detailed comments on the notes for chapters 1 – 9 are the chief reason there are so few corrections for those chapters. (The exercises on page 130 were added later!)

### Corrections to “Analysis, an Introduction”

p. 6, eqn. (14):  $(m-n)+(p-q)=(m+p)-(n+q)$

p. 7, line 9: ... and satisfies M1, M2,

p. 26, exercise 2: (ii)  $1, x, x^2, \dots, x^n$  are linearly dependent over  $\mathbb{Q}$ .

exercise 2: (iv) there are real numbers  $y_1, y_2, \dots, y_n$  such that ...

p. 28, line 16:  $= |z|^2 + 2\operatorname{Re}(z\bar{w}) + |w|^2 \leq \dots$

p. 66, exercise 4: right side should be  $1/(1-z)^{k+1}$ .

exercise 5: Under the given assumptions, show there is a power series  $\sum_{n=0}^{\infty} b_n z^n$  with radius of convergence at least  $R-r$  such that

$$\sum_{n=0}^{\infty} a_n z^n = \sum_{n=0}^{\infty} b_n (z - z_0)^n, \quad |z - z_0| < R - r.$$

p.98, line 2:  $n > \|f\|/(\delta^2 \varepsilon)$

p. 103, line 3:  $= f'(c)[g(b) - g(a)] - g'(c)[f(b) - f(a)];$

line 4: divide by  $[g(b) - g(a)]g'(c)$ .

p. 115, lines 1-2: Suppose that  $f$  is a real-valued function ...

p. 124, equation (14):  $z^n + a_{n-1}z^{n-1} + \dots$

p. 130, exercise 8:  $F(a, b, c; z)$

exercise 9:  $F(a, b, b; z)$

exercise 10:  $\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tz)^{-a} dt$

p. 137, line 6:  $\dots + m^*((E \cap (A \cup B)) \cap A^c)$

p. 149, line 8: ... for  $x \in [0, 1] \cap C^c$  ...

p. 152, line 13:  $\int \int (f + g)$  should be  $\int (f + g)$

p. 170, line 9:  $\{h^* > \delta/2\}, \quad \{h > \delta/2\}$

p. 171, line 5: ... and define a subfamily  $\mathcal{I}_{n+1}$  ...

line 6: ... if  $\mathcal{I}_{n+1}$  is empty ...

line 11: ... implies that  $|I_k| \geq |I|/2$  ...

line -10: ...  $I \subset I_n^*$  for some n.

p. 178, equation (17): omit  $1/2\pi$

p. 181, line -9:  $\sum_{n=0}^{2N} (e^{ix})^n$

- line -11: ... as long as  $e^{ix} \neq 1$ ,
- p. 183, line 9:  $S_N f(x_0) - f(x_0) = \dots$   
line 10: ...  $[f(x_0 - y) - f(x_0)] dy$   
equation (31):  $\sin\left(\left[N + \frac{1}{2}\right] y\right)$
- p. 186, line -5,-4: ... property (ii) in Proposition 13.9 ...  
line -2:  $\int_{|y| < \delta}$   
line -1:  $\int_{\delta < |y| < \pi}$
- p. 189, line -4: See Exercise 9 of Section 12D.
- p. 192, line 8:  $f_N(x) = \sum_{-N}^N a_n e^{inx}$
- p. 192, line -2:  $a_0 = 0$
- p. 193, line 3: Use Exercise 5 of this section and ...
- p. 194, exercise 10:  $f(x_0-) = \lim_{x \rightarrow x_0, x < x_0} f(x)$ ; ...
- p. 196, exercise 4:  $g_n(x) = \dots$
- p. 201, equation (6):  $0 \leq t \leq 2\pi$
- p. 211, line 5: ... let  $I_L = [-\frac{1}{2}L, \frac{1}{2}L]$  ...  
line 8:  $\int_{-L/2}^{L/2}$
- p. 214, exercise 6:  $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$
- p. 215, exercise 9: ... (assume (46) and use Dominated Convergence).
- p. 217, equation (57):  $V_T = \|(T - E_T I)\psi\|^2$
- p. 227, line 11: ... but it may not be unique.
- p. 234, line 1:  $\mathbf{f}(x, \mathbf{y})$
- p. 238, proof of Lemma 238. It might seem at first that the sets removed from  $\tilde{A}_2$  and  $\tilde{A}_3$  to form  $A_2$  and  $A_4$  are empty, but there is a nonzero vector that is fixed by  $\sigma$ .
- p. 239, statement of Theorem: there should be four  $B'_j$  and three  $C'_j$ : drop  $C'_4$  and take
- $$B'_j = B_j \cap (D_f \cup D_\infty), \quad j = 1, 2; \quad B'_j = B_{j-2} \cup D_g, \quad j = 3, 4.$$
- Then the first four  $A'_j$  correspond to the  $B'_j$  and the last three to the  $C'_j$ .
- Also:  $f : D_\infty \rightarrow A_\infty$  and  $g : A_\infty \rightarrow D_\infty$  are bijective (but not inverses of each other).
- p. 243, Section 3A: Omit 12, and renumber 13 – 17 as 12 – 16.
- p. 251, Section 13B, 6 (b): ... the interval  $[b, b + 2\pi)$  ...
- p. 252, Section 13G: 1, 2. Use Theorems 13.12 and 13.14 and Proposition 13.4. Note that a continuous periodic function is an  $L^2$ -periodic function. For exercise 1 (b), continuity or lack of continuity can be seen by comparison with a multiple of the square wave function in §14A.