Problem 1. Determine whether the following improper integrals converge or diverge. For the integrals that converge, find the value.

1. \( \int_0^1 x^{-\frac{2}{3}} \, dx \).
2. \( \int_0^1 x^{-\frac{3}{2}} \, dx \).
3. \( \int_0^\infty e^{-x} \, dx \).
4. \( \int_1^\infty (x^{-\frac{1}{3}} - x^{-\frac{1}{2}}) \, dx \).
5. \( \int_0^\pi \tan^2(x) \, dx \).

Problem 2. Find \( \int_1^\infty \frac{\log(x)}{x^2} \, dx \).

Problem 3. Find an anti-derivative \( \int \frac{dx}{x \log(x)} \).

Then, using a calculator, find \( \int_1^{100} \frac{dx}{x \log(x)} \) to a few decimal places.

Does \( \int_1^\infty \frac{dx}{x \log(x)} \) converge or diverge?

Problem 4. Find \( \int_0^\infty \frac{dx}{x^2 + 3x + 2} \).