408L CLASS PROBLEMS

MARCH 11TH, 2020

Problem 1. Find f_x and f_y for $f(x,y) = \frac{x^2y^3}{x+y^2}$.

Solution. We have:

$$f_x = \frac{(x+y^2)*2xy^3 - x^2y^3*1}{(x+y^2)^2} = \boxed{\frac{x^2y^3+2xy^5}{(x+y^2)^2}}$$

by the quotient rule, and similarly:

$$f_y = \frac{(x+y^2)*3x^2y^2 - x^2y^3*2y}{(x+y^2)^2} = \boxed{\frac{3x^3y^2+x^2y^4}{(x+y^2)^2}}.$$

Problem 2. Find f_x and f_y for $f(x,y) = \frac{xy}{\sin(x)+\cos(y)}$.

Solution. We have:

$$f_x = \frac{(\sin(x)+\cos(y))*y - xy*\cos(x)}{(\sin(x)+\cos(y))^2} = \boxed{\frac{\sin(x)y+\cos(y)y - xy\cos(x)}{(\sin(x)+\cos(y))^2}}$$

and:

$$f_y = \frac{(\sin(x)+\cos(y))*x + xy*\sin(y)}{(\sin(x)+\cos(y))^2} = \boxed{\frac{\sin(x)x+\cos(y)x + xy\sin(y)}{(\sin(x)+\cos(y))^2}}$$

Problem 3. Find f_x and f_y for $f(x,y) = 11x^3 + y^2 - 11xy - 2y$. Evaluate $f_x(2,12)$ and $f_y(2,12)$. What might your answer tell you about the function $f(x,y)$ near $(2,12)$?

Solution. We have:

$$f_x = 33x^2 - 11y$$
$$f_y = 2y - 11x - 2.$$

Therefore, $f_x(2,12) = 33*4 - 11*12 = 0$ and $f_y(2,12) = 2*12 - 11*2 - 2 = 0$.

As in 1-variable calculus, this suggests that $f(x,y)$ may have a local minimum or maximum at $(2,12)$. (In fact, it is a local minimum.)

(To be clear: we are not testing you on this material. But I still want you to be vaguely aware that partial derivatives can be used to test extrema for functions in several variables.)

Problem 4. A function $f(x,t)$ describes a wave at a position x and time t if it satisfies the *wave equation*:

$$f_{xx} = f_{tt}.$$

Which of the following functions satisfies the wave equation?

(1) $f(x,t) = \cos(x)\sin(t)$.
(2) $f(x,t) = e^{xt}$.
(3) $f(x,t) = e^{x+t}$.
(4) $f(x,t) = \sin(x+t)$.
(5) $f(x,t) = \log(\frac{x+t}{x-t})$.

Solution. All of these functions satisfy the wave equations except for the second.
The trickiest one to check is $f(x,t) = \log(\frac{x+t}{x-t})$. In fact, we have:

$$\log(\frac{x+t}{x-t}) = \log(x+t) - \log(x-t).$$

We then have:

begingathered f$_x = \frac{1}{x+t} - \frac{1}{x-t}$
$f_{xx} = \frac{-1}{(x+t)^2} + \frac{1}{(x-t)^2}$

and:

$$f_x = \frac{1}{x+t} - \frac{1}{x-t}$$
$$f_{tt} = -\frac{1}{(x+t)^2} + \frac{1}{(x-t)^2}$$

giving the claim.