408L CLASS PROBLEMS

APRIL 13TH, 2020

Problem 1. Does $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^2}$ converge absolutely, converge conditionally, or diverge?

Solution. By the p-series test, $\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges, so the series $\boxed{\text{converges absolutely}}$.

Problem 2. Does $\sum_{n=0}^{\infty}(-1)^n\frac{10^{100}n}{n^2+1}$ converge absolutely, converge conditionally, or diverge?

Solution. By the alternating series test, the series converges. However, by applying the comparison test with the harmonic series, the test $\sum_{n=0}^{\infty}\frac{10^{100}n}{n^2+1}$ diverges. Therefore, the series $\boxed{\text{converges conditionally}}$.

Problem 3. Does $\sum_{n=2}^{\infty}(-1)^n\frac{1}{n\log(n)}$ converge absolutely, converge conditionally, or diverge?

Solution. By the alternating series test, the series converges. We have seen previously that the integral test implies that $\sum_{n=2}^{\infty}\frac{1}{n\log(n)}$ diverges. Therefore, the series $\boxed{\text{converges conditionally}}$.

Problem 4. Does $\sum_{n=0}^{\infty}(-1)^n\frac{n!}{2^n}$ converge absolutely, converge conditionally, or diverge?

Solution. We have:

$$\frac{n!}{2^n} = \frac{n}{2}\cdot\frac{n-1}{2}\cdot\ldots\cdot\frac{2}{2}\cdot\frac{1}{2}.$$

Clearly this sequence diverges to ∞, so the series $\boxed{\text{diverges}}$.