408L CLASS PROBLEMS

JANUARY 29, 2020

Problem 1. Find $\int \cos(x)e^{\sin(x)}dx$.

Solution. We set $u = \sin(x)$, so $du = \cos(x)dx$. We then have:

$$\int \cos(x)e^{\sin(x)}dx = \int e^{\sin(x)} \cdot (\cos(x)dx) =$$
$$\int e^u du = e^u + C = \boxed{e^{\sin(x)} + C}.$$

Problem 2. Find $\int \frac{x^3}{\sqrt{1+x^4}}dx$.

Solution. We set $u = 1 + x^4$. Then $du = 4x^3dx$, or equivalently, $x^3dx = \frac{1}{4}du$. We therefore have:

$$\int \frac{x^3}{\sqrt{1+x^4}}dx = \frac{1}{4}\int \frac{1}{\sqrt{u}}du = \frac{1}{4}\int u^{-\frac{1}{2}}du =$$
$$\frac{1}{2}u^{\frac{1}{2}} + C = \boxed{\frac{1}{2}\sqrt{1+x^4} + C}.$$

Problem 3. Find $\int \frac{x}{1+x^4}dx$.

Solution. Let $u = x^2$, so $du = 2xdx$. We then have:

$$\int \frac{x}{1+x^4}dx = \frac{1}{2}\int \frac{1}{1+u^2}du = \tan^{-1}(u) + C = \boxed{\tan^{-1}(x^2) + C}.$$

Problem 4. Find $\int \frac{\cos(x)}{2-\cos^2(x)}dx$.

Solution. We see $\cos(x)dx$, so think to use $u = \sin(x)$. To make this reasonable, we use the trig identity:

$$\sin^2(x) + \cos^2(x) = 1 \Rightarrow \cos^2(x) = 1 - \sin^2(x)$$

to write:

$$\int \frac{\cos(x)}{2-\cos^2(x)}dx = \int \frac{\cos(x)}{1+\sin^2(x)}dx.$$

Using the u-substitution above ($u = \sin(x)$, $du = \cos(x)dx$), we then obtain:

$$\int \frac{\cos(x)}{1+\sin^2(x)}dx = \int \frac{1}{1+u^2}du = \tan^{-1}(u) + C = \boxed{\tan^{-1}(\sin(x)) + C}.$$