

408L CLASS PROBLEMS

APRIL 24TH, 2020

Problem 1. Show that $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$. Using the first two terms in this Taylor expansion, estimate $\sin(.01)$ to 8 decimal places.

Problem 2. Using Taylor's theorem, find the Taylor series of $\frac{1}{1-x}$ about $x = 0$.

Problem 3. Find the first three terms of the Taylor series of $\sqrt{1+x}$ about $x = 0$. Using these, estimate $\sqrt{1.1}$.

Problem 4. Find the Taylor series for $\tan^{-1}(x)$. Then evaluate the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$.