408L CLASS PROBLEMS

FEBRUARY 10TH, 2020

Problem 1. Find the volume of the solid by rotating the area under the graph of $\sqrt{x} \cdot e^{x^2}$ for $0 \leqslant x \leqslant 1$ around the x-axis.

Problem 2. Find the volume of the solid obtained by rotating the area between the x-axis and the graph of $\cos(x)$ for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$ about the line $y = 1$.

Problem 3. In this question, we find the area of a circle of radius 1. (Of course you know the answer, but the point of the problem is to calculate it yourself.)

(1) Using the cross-section method, show that the area of a circle with radius 1 is:

$$2\int_{-1}^{1} \sqrt{1-x^2}dx.$$

(2) Evaluate this integral. (Hint: use u-substitution with $u = \sin^{-1}(x)$.)

Problem 4. Find the area lying above the line $y = \frac{1}{\sqrt{2}}$ and within a circle of radius 1 about the origin.