408L FIRST MIDTERM REVIEW

FEBRUARY 21, 2020

Subjects we have covered so far:

- Integration by parts
  - Tips: if you see $\int t^2 e^t dt$, good idea to take $u = t^2$. I.e., $u = t^n$ is often a good idea.
  - Make sure you know how to integrate dv. Often, if you see a factor you know how to integrate, good idea to choose $dv =$ that factor. For example, $\int t^2 e^{t^2} dt$, recognize that you know how to integrate $te^{t^2} dt$, so you take $dv = te^{t^2} dt$.
  - If $\int f(x)dx$ and $f(x)$ is complicated but has a simpler derivative, good idea $dv = dx$ and $u = f(x)$. E.g., works for $\int \ln(x)dx$ and $\int \arctan(x)dx$. Really, always a good idea to take $u = \ln(x)$ or $u = \arctan(x)$ if they appear in the problem.
  - Try Googling "lixet integration by parts" (or lipet?) for additional tips.
  - Practice.
- u-substitution
- Fundamental theorem of calculus
  - $\int_a^b f(x)dx = F(b) - F(a)$ for $F(x)$ an anti-derivative of $f(x)$, i.e. a function with $F' = f$.
  - One problem type: $F(x) = \int_a^x f(x)dx$, find the derivative of $F(x)$. (It's $f(x)$.)
  - Maybe more complicated: $G(x) = \int_{\sqrt{x}}^{x^2} f(x)dx$. Find the derivative of $G(x)$.

    To do this: $F(x)$ is an anti-derivative of $f(x)$, so $G(x) = F(x^2) - F(\sqrt{x})$. Know: $F'(x) = f(x)$. Apply the chain rule.
- Recognizing even/odd functions.
  - Key sign: $\int_{-10}^{10} f(x)dx$, or something like that.
- Area between curves.
- Volumes, especially of rotation.
  - Key idea: volume $= \int$(Area of a cross section)dx.
  - Washer method for solids of rotation.
- Trigonometric substitution.
- Trigonometric integrals.
- Indefinite/definite integrals.

Trigonometry:

- Double angle identities: $\cos(2x) = \cos^2(x) - \sin^2(x)$, $\sin(2x) = 2\sin(x)\cos(x)$.
- Half angle identities: $\cos^2(x) = \frac{1+\cos(2x)}{2}$, $\sin^2(x) = \frac{1-\cos(2x)}{2}$.

  - Used for integrating $\cos^2(x)$ and $\sin^2(x)$.
- $\cos^2(x) + \sin^2(x) = 1$.
- $1 + \tan^2(x) = \sec^2(x)$.
- Integrals and derivatives of all basic trig functions. (Hardest ones: $\int \sec(x)dx$ and $\int \csc(x)dx$.)
- Inverse trig functions and their derivatives.
  - $\int \frac{1}{1+x^2}dx = \arctan(x)$.
  - $\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin(x)$.
- Everything can be expressed in terms of sin and cos: $\tan(x) = \frac{\sin(x)}{\cos(x)}$, $\sec(x) = \frac{1}{\cos(x)}$, $\csc(x) = \frac{1}{\sin(x)}$, $\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$. (Helpful if you don't see a right answer on the multiple choice list.)

Fundamental integrals to know:

- $\int x^n dx = \frac{x^{n+1}}{n+1}$.
- $\int \sin(x)dx = -\cos(x)$, $\int \cos(x)dx = \sin(x)$.
- $\int \frac{dx}{x} = \ln(x)$.
- $\int e^x dx = e^x$.
- Integrals/derivatives of all trig functions.