18.786 References

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Quadratic extensions of local and global fields

Jean-Pierre Serre, *A course in arithmetic*, vol. 7 (Springer Science & Business Media, 2012).

The Hilbert symbol is the main subject of Part I of this book, especially Chapter III. It only treats completions of $\mathbb{Q}$, but the material generalizes away from $\mathbb{Q}_2$ to general local fields. The book emphasizes elementary techniques, so can give a feeling for what is going on in a more abstract framework.

Local class field theory

Jean-Pierre Serre, *Local fields*, vol. 67 (Springer Science & Business Media, 2013).

A classic reference that rewards the effort you put into it. It begins with the structure theory of local fields, develops group cohomology from scratch, and then proves the main theorem of local class field theory. Unfortunately, this book does not do the work of plainly laying bare its main threads, so requires some patience for self-study.

Ivan B. Fesenko and Sergei V. Vostokov, *Local fields and their extensions*, vol. 121 (American Mathematical Soc., 2002).

A newer reference, with updates on the developments of the subject since Serre. Very detailed, with many exercises. Conveniently available online at `https://www.maths.nottingham.ac.uk/personal/ibf/book/vol.pdf`.

Class field theory (local and global)

Emil Artin and John Torrence Tate, *Class field theory*, vol. 366 (American Mathematical Soc., 1967).

An original source for many of the ideas of global class field theory. Unfortunately, it does not treat local class field theory.

J.W.S. Cassels and Albrecht Fröhlich, *Algebraic number theory.* (Academic Press, Thompson Book Co., 1967).

Notes from an old conference, developing the whole theory more or less from scratch. Notes available from many different authors. The quality sometimes varies, but is often high. In particular, Serre and Tate contribute the notes on local and global class field theory respectively, and generally speaking, anything written by either of them is required reading.

André Weil, *Basic number theory.*, vol. 144 (Springer Science & Business Media, 2013).

Another classic text. It gives a cohomological treatment of class field theory without every saying the words, which is both a bug and a feature. It is the only source I know with a detailed approach to the proof of the main global theorem via zeta functions.

James S Milne, "Class field theory," Available at `http://www.jmilne.org/math/CourseNotes/CFT.pdf` (2013).

These pleasantly written notes, which cover the subject in detail, are a solid reference for most of the ideas of class field theory.

Texts by Neukirch

Neukirch, who was an exemplary expositor, wrote *two* books with the same name:

Jürgen Neukirch, *Class field theory*, vol. 280, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences] (Springer-Verlag, Berlin, 1986)

and:

Jürgen Neukirch, *Class field theory*, The Bonn lectures, edited and with a foreword by Alexander Schmidt, Translated from the 1967 German original by F. Lemmermeyer and W. Snyder, Language editor: A. Rosenschon (Springer, Heidelberg, 2013).

The former is really a geodesic approach to the subject that minimizes the role of group cohomology, only using it in the case of cyclic groups, where it is more elementary.

Two other relevant books, one less advanced and one more advanced than the present course:

Jürgen Neukirch, *Algebraic number theory*, vol. 322 (Springer Science & Business Media, 2013)

is a great introduction for the general background on number fields, and:

Jürgen Neukirch, Alexander Schmidt, and Kay Wingberg, *Cohomology of number fields*, vol. 323 (Springer Science & Business Media, 2013)

is a more advanced treatment of Galois cohomology and its role in arithmetic.

Zeta functions

B Riemann, "On the number of primes less than a given magnitude," *Monthly Reports of the Berlin Academy* (1859)

Still the classic reference, and definitely worth a read. English translations are readily available.

John Torrence Tate, "Fourier analysis in number fields and Hecke's zeta-functions" (PhD diss., Princeton University Princeton, NJ, 1950)

Tate's thesis, which was reprinted in Cassels-Froöhlich. Its major contribution is to reinterpret Riemann's work on the analytic properties of the zeta function by using Fourier analysis not on $\mathbb{R}/\mathbb{Z}$, but on $\mathbb{A}_{\mathbb{Q}}/\mathbb{Q}$. This is very useful for generalizing to number fields (c.f. to the treatment of Hecke's work in Neukirch's *Algebraic number theory*), and much more clearly highlights the mechanisms underlying the analytic theory.

Homological algebra

Sergei I. Gelfand and Yuri I. Manin, *Methods of homological algebra* (Springer Science & Business Media, 2013)

A great place to learn the subject. Different people tend to take different things away from it, which is a great sign of its richness. It gives a heavy emphasis on triangulated categories, and I personally think it would be improved by focusing more on chain complexes themselves.

Alexandre Grothendieck, "Sur quelques points d'algèbre homologique," *Tohoku Mathematical Journal, Second Series* 9, no. 2 (1957): 119–183

There's a lot that's great about this paper still, but also a lot that's dated. For example, I find the ∂-functors story to be dated (and uninspiring).

Henri Cartan and Samuel Eilenberg, *Homological algebra*, vol. 19 (Princeton University Press, 1999)

Another old text, with similar deficiencies. But it has many virtues, and is filled with examples. Much of the subject was invented here, which also makes it a great source from which to learn.