

TOPICS IN NUMBER THEORY: COMMUTATIVE FORMAL GROUPS

INSTRUCTOR: ZHIWEI YUN

1. PREREQUISITE

Abstract algebra (groups, rings, modules, Galois theory);

Local fields (knowledge of local class field theory is preferred but not strictly required);

Algebraic geometry (Hartshorne Ch. I-IV; better if you took Math 216A,B,C).

Note: if you have not learned schemes seriously but meet the first two requirements, you should be able to follow half of the course.

2. TOPICS TO BE COVERED

We have 20 classes in total. Each of the following three topics will roughly take 6 classes.

1. One-dimensional formal groups: structure theory of Lazard; Lubin-Tate groups and application to local class field theory; relation with elliptic curves. [15], [13]

2. Cartier theory. [28], [16]

3. p -divisible groups; Dieudonné-Manin theory. [25], [1], [17]

If there is more time, we have the following topics “in reserve”: Drinfeld modules, formal groups and topology, moduli of formal groups, applications to abelian varieties...

3. TENTATIVE SCHEDULE

Sep. 25	Introduction; Basic examples; Logarithms
Sep. 27	Fundamental results for 1-dimensional formal groups
Oct. 2	Classification by heights
Oct. 4	Formal A -modules
Oct. 9	Lubin-Tate groups
Oct. 11	Explicit local class field theory
Oct. 16	Formal groups and cohomology theories (Lurie)
Oct. 18	Formal groups as functors
Oct. 23	Cartier ring
Oct. 25	Witt ring
Oct. 30	Cartier's theorem
Nov. 1	p -typicality
Nov. 6	p -typical Cartier theory
Nov. 8	Finite flat group schemes
Nov. 13	p -divisible groups
Nov. 15	p -divisible groups
Nov. 27	Dieudonné-Manin theory
Nov. 29	Dieudonné-Manin theory
Dec. 4	One of the topics in reserve
Dec. 6	One of the topics in reserve

4. REFERENCES

Comprehensive references: [16], [6], [28], [17]

p -divisible groups: [25], [22], [3], [1]

Extended reading: moduli [14], [9], [4]; Relation to topology [21]; Drinfeld modules [2]

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